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Tool-mediated zone of proximal development

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Context

In this short study, I offer to describe mathematical tools as any tool-like object used to solve a mathematical task. Investigating the interrelationships between the physical properties of some "Cuisenaire rods" and children's knowing of fractions, I ask : Can a zone of proximal development emerge from the guidance provided by the tools?

Introduction

I describe mathematical tools as any tool-like object used to solve a mathematical task. The *tool-like* object refers to Marx's view of the use of working tools; where man uses the physical and mechanical properties of objects to reach his goals (Marx & Engels, 1965). And, the meaning I give to *object* is derived from Engeström's (2009) conceptualisation of *object* as any focus of attention. Thus, for the purpose of this study, my emphasis on *tool*-like object is, the following: I consider both x^2 and an abacus as mathematical objects, yet an abacus is a mathematical tool and x^2 is not a mathematical tool because we don't use the physical or mechanical properties of the sign. I refer to the characteristics of the tools as their physical properties such as rigidity, shape, and sharpness. Therefore, a tool becomes a *mathematical* tool only if a person uses its physical property to think about mathematics or to solve a mathematical task. Finally, the term "perception" is used to refer to the aspects of a child's thinking that contribute to the kinds of interaction that happen. Hence, a paper clip can become a mathematical tool if a child who uses its physical properties (e.g., its size or rigidity), for example, to realize some measurement.

Although mathematical tools are conceptualised as being useful, I argue that their usefulness depends on:

- their physical (or mecanical) properties;
- the child's *perceptions/knowing* while interacting with it; and
- the task at hand

In this reflection, I investigate the interrelationships between the physical properties of the "Cuisenaire rods" mathematical tools, and children's knowing of fractions. The question guiding my thinking is: Does a zone of proximal development (ZPD) emerge from the guidance provided by the rods, i.e. the physical properties of the mathematical tools that helps the children in solving fractional problems?

My reference to ZPD is through Vygotsky's description as being "the distance between the actual developmental level (independent problem solving) and the level of potential development (problem solving under adult guidance or in collaboration with more capable peers)" (1978, p. 69). More specifically, I employ Roth and Radford's (2010) participative conceptualisation of the ZPD, as "the emergence of a new form of collective consciousness, something that cannot be achieved if we act in solitary fashion" (p. 306) in order to examine the contribution of a tool (Cuisenaire rods). Following Vygotsky's view, in the field of mathematics education the more knowledgeable others are conceptualised as agents such as teachers, adults and peers. Furthermore, the

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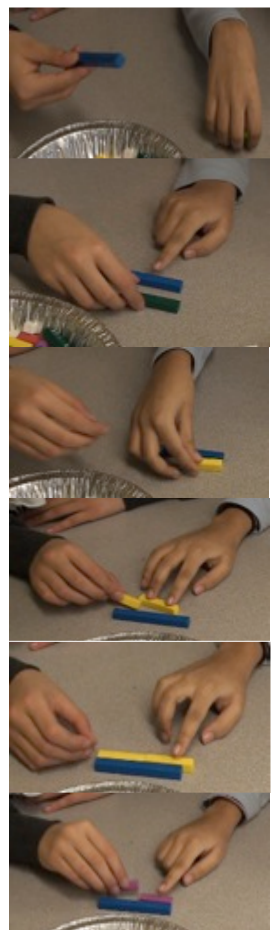
communication and interactions that take place within the ZPD usually have been referred to as sign-mediated and inter-subjective (Lerman & Meira, 2001; Roth & Radford, 2010), meaning that the ZPD is looked at between children and adults or amongst children themselves. Here, I proposed to extend the interactions within the ZPD to include *tool-mediation* (as viewed by Marx, and thus not only about signs), and extend the notion of the 'more knowledgeable other' to include tools as well, with the resources provided by their physical (or mechanical) properties (e.g. the ones used by children in mathematics classrooms).

My intentions are to see how a ZPD emerges as children interact with a mathematical tool, and reflect on the role of the tool in that situation by asking who/what is a more knowledgeable other? Drawing on a few episodes involving elementary school student, I argue that in children's interactions with the mathematical tools there is a complex relationship between the mathematical knowing of the children and (change in) their use of the mathematical tools. Roth and Radford's (2010) described *knowing* as "the possibilities that become available to the participants for thinking, reflecting, arguing, and acting in a certain historical and cultural practice" (p. 301). Therefore, in the case reported here for this reflexion, the historical and cultural practice would be the practice of adding two fractions.

Some Analyses

With the following fragment, I examine two grade 7 children's participations in an interaction with Cuisenaire rods in order to add two fractions. These two children did not have any prior experience with the Cuisenaire rods. Moreover, at the time of learning the concept of the addition of fractions, these children did not use any mathematcial tool. I argue that in the follwowing interaction, the children, in multiple attempts, tried to perceive the interlealtionship between the sizes of the Cuisenaire rods or their colours) to represent ½ and 2/5. These two children use different rods as their unit and then use the interrelationship between the pieces in terms of their sizes and relative lenghts, to figure which rods could present ½ and which rod could present 2/5.

The following excerpt of trancript shows how they first chose the blue rod as their whole one unit and then consider different rods according to their sizes as possiblilities to represent the fractional amount of ½ with respect to the length of the blue rod as a whole:



A: [Picks up the blue rod of 9]

M: I guess this is one

A: [Picks up the dark green rod of 4]

M: This isn't really...

A: [Puts the dark green rod back]

A: [Picks up the yellow rod of 5]

M: Okay then this could be half, so...

A: [Picks up another yellow rod of 5]

A: [arrange the two rods of 5 underneath the blue rod]

M: No, not quite a half

A and B: [Pick up the pink rod of 3]

A: Both [arrange the two rods of 3 underneath the blue rod of 9]

M: No...

The students' actions suggest that they first perceived the lenghts of the rods as a way to represent the fractions: the green rod, then the

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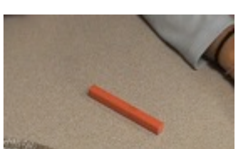
yellow rod, then the pink rod where used as possible half of the blue rod. Here, it is the physical properties of the rods that assisted them in solving the problem, in the sense that they could compare their sizes and see that none of these rods could represent ½ of the blue.

In this, the children collectively reached the interactional achievement through their interactions with the rods, becoming acquainted with a different form of expression, action in relation to the whole unit and ½ the unit. Example of such expressions and actions include children's movement of different pieces as the attempted to find half a unit: they picked *[the yellow rod of 5]* or they arranged *[the two rods of 5 underneath the blue rod]* or they stated: "No, not quite a half", and so on. These forms of expressions and actions connects with important, meaningful mathematical ideas in relation with representing, comparing, or measuring for example, and they have emerged in the fragment as the students used rods in various ways, thanks to their material propreties (shape, color, composition).

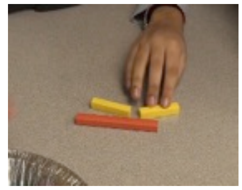
Moreover, I want to argue, by using the new fragment, that the interrelationship between the sizes of the rods can guides the students through *new* forms of reflecting, talking and acting.

In the episode taking place a little bit later, the students change their "one" unit for the orange rod of 10, and again use the physical properties of the rods to find which rod could be its half and, and which rod could be its 2/5. The following excepts of transcript shows this interaction:



A: This could be one [picks up an orange rod of ten]

Both: [puts two dark green underneath it]



M: [puts two yellow underneath it]
A: Okay
M: Yeah okay that is good
A: Yeah
M: So we need one of these
A: And then two fifths

A:[puts two lime green underneath the orange]
A: No... that is three

M: [pick up the red rod]
M: The red

Both: [Line up the reds underneath the orange rod]

M:[puts rod red rods next to the yellow rod]

In this interaction, the students perceived the length of the orange rod as a potential referential unit ("This could be one"). Then they seemed to similarily perceive the sizes of the green and the yellow rod being half of the orange rod. Then, the guidance provided by the physical properties of the rods assisted them to select the yellow rod to represent the ½ of the orange rod. The next interaction was for the students to find the rod that was 1/5 of the orange rod. They first tried the lime green rods (and stated "No... that is three"), and then try with the red rods. They then again used the physical properties of the rods to perceive how the red rods is 1/5 of the orange, as 5 red rods fit underneath the orange one:

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A: Okay, these are fifths, how many of those?
M: Two
A: Two fifths

In this interaction between the children and the rods, a new form of reflecting on and expressing about the concept of fraction has emerged as the children participated in a collective interaction with the Cuisenaire rods. I argue for new form of expression and action, because previously these children never represented rational numbers using Cuisenaire rods, and mostly knew these numbers (especially one like 2/5) through their mathematical symbolization. Their interactions with the tools made it possible for the children to express fifths and half of a unit not as a mathematical symbol but as interrelation between pieces of the Cuisenaire rods (see Figure 1). Evidence of this new understanding is easilly observable in the remaining of the episode and in (these and others) children's actions in subsequent lessons, where students use Cuisinaire rods (and their relative lenght) to solve other problems.

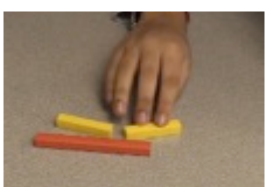

Figure 1: Expressing half a unit not as an interrelation

Through the above-mentioned interactions, I showed how children started using the Cuisenaire rods, not knowing how the properties of the rod could be useful in representing fractions (i.e., 1/2 and 2/5). Later, as children got aquineted with the properties of the Cuisenaire rods, in terms of the relationships between their sizes, they were then able to use the physical properties of the tools to represent the fractions.

That is, I propose that the children's learning occurred through the emergence of a ZPD under the guidance provided by the physical properties of the Cuisenaire rods. In this case, *the more knowledgeable other was the tool, thanks to its physical property*. To achieve this, they evidently draw on their previous knowledge (that is, action possibilities) of how to manipulate objects to take advantage of the length in measurement (neat alignments), and wisely choose the next rod to "try out" (using smaller ones each time). But again, it is important to note that it is the *actual* interrelationship between the sizes of the rods that guided them.

The general issue that I underline here is how a ZPD emerges as children interact with mathematical tools, thanks to the physical properties of the tools now acting as the more knowledgeable other. In the vygotskian tradition, tools (including mathematical tools) are culturally and historically based (Wertsch & Rupert, 1993), and their design socially originate in human activity. Therefore I suggest that more knowledgeable-other-ness of the tools is based on and originated from the many others who have, overtime, designed, used and modified the tools.

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