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RIGHT TO MATHEMATICS: FIVE ISSUES

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Is mathematics a fundamental human right? Derrida's exploration of philosophy prompts us to consider similar questions for mathematics. Drawing on concepts used by Derrida to highlight fundamental issues in the context of the right to philosophy, I examine the relationship between mathematics and human rights. Applying Derrida's analysis calls for mathematically rich environments as a fundamental right, which entails not only the right to engage in mathematical activity but also a redefinition of mathematics itself to legitimize diverse forms of participation.

DERRIDA'S RIGHT TO PHILOSOPHY

In *Right to Philosophy*, Derrida (1990, 1994) collected some of his writing about the teaching of philosophy, the academic institution, and the politics of philosophy in educational systems. His work raises important questions, such as: What is philosophy: What could be *a* or *the* "right" philosophy? How is it made accessible: Could we go "right" to it? What could be the "rights and responsibilities" regarding to doing it "right"? With this, Derrida patiently deconstructs various hierarchies, for instance, between writing and speech, self and other, presence and absence, and thereby challenges (and disrupts) what can be seen as systems of domination and oppression. Embedding these questions in philosophy itself, the "right to philosophy" imagined by Derrida thus highlights, but also problematizes, its liberatory power.

In this study, the first question we ask is: Could we reproduce Derrida's analysis to explore similar questions in relation with mathematics and mathematics education? From Derrida's deconstruction of various interpretations of the "right to philosophy", we can extract five concepts highlighting fundamental issues in the context of the right to philosophy: Pharmakon, Logocentrism, Différance, Metaphysics of Presence and Arche-writing

Pharmakon, Derrida explains, is a concept derived from an ancient Greek word that can refer to a substance that is both (or neither) a remedy and/or a poison. The idea is to exhibit binary oppositions to examine the paradoxical nature of concepts, practices, and institutions. *Pharmakon* symbolizes the dual nature of philosophy itself—potentially exclusionary (poisonous) due to how complex and demanding it is, but potentially helpful to human life (a remedy), nurturing critical thinking and intellectual growth. Philosophy assumes that all human beings have the right and the ability (even the obligation, the requirement, the responsibility, etc.) to think, but also perpetuates elitism, favoring specific academic backgrounds, methodologies, perspectives (e.g. de Beauvoir first faced dismissal and even ridicule). Some questions are then: Who has access to philosophy? Who philosophizes?

The above connects with *logocentrism*, a philosophical orientation that emphasizes the importance of speech and written words as forms of meaning. It is used to show tensions between what takes place here and now versus ancient, traditional, or external activities (for example, scholars or ancient philosophers, versus contemporary everyday actors and situations). Challenging logocentrism means calling for diverse voices and perspectives to engage with philosophy, question what it means to do philosophy here and now, and why philosophy "as a discipline" needs to be updated. Here, we can think, for example, about how non-Western philosophies (including oral traditions) are often characterized by marginalization, ethnocentrism, neglect, and so on.

Différance conceptualizes meaning as disrupting, and challenges essentialist and foundationalist assumptions about stability and certainty. It conveys the idea that meaning and identity are not fixed nor stable, but are instead deferred and differentiated through a process of ongoing interpretation and reinterpretation. Highlighting deferral and difference in meaning implies emphasizing how multiple interpretations and understandings coexist; a diversity of analyses and viewpoints should be acknowledged and respected. Beyond that, *différance is* also concerned with how ideas/understanding transform over time. In philosophy, great examples of this are de Gobineau's famous "Essay on the Inequality of the Human Races" (first widely accepted, now largely rejected) and Nietzsche ideas (once scorned, now considered among the most influential in Western intellectual history).

Metaphysics of Presence is a way to describe (and contest) the assumed transparency and immediacy associated with the notion of presence, and to question the idea of "representation" and how truth might be "exposed". Ambiguity, indeterminacy, and instability then make room for what is (always) missing, or absent. Derrida particularly draws attention to the performative aspect of language, how philosophy is something we "do" in language, and how language is always ambiguous, evocative, unstable, incomplete (we never perfectly say (all of) what we mean, and vice-versa).

Finally, the concept of *arche-writing* underscores how philosophical ideas are not confined to a specific medium or tradition, but part of an ever-growing landscape, and interconnected tissue of communication systems (e.g. of traces) that transcend cultural boundaries. Derrida insists on the importance of surrounding contexts, discourses, and traditions. Showing, for instance, the intricate relationship between philosophy and politics (political structures, ideologies, etc.), he emphasizes the importance of critically engaging with political issues (e.g. how UNESCO shapes and is shaped by broader socio-political dynamics, values, and practices that guide international collaborations). *Arche-writing* seeks to destabilize the distinction between origin and derivation, and draws attention to the conditions for the process of meaning-making to occur, thus coherently questioning language, for example, as a dynamic and fluid system.

In the next section of this paper, I re-interpret these concepts in relation with mathematics and mathematics education. Revisiting *pharmakon*, *logocentrism*, *différance*, *metaphysics of presence* and *arche-writing* separately in this way is difficult and somewhat artificial: In Derrida's writing, they all appear together, completely intertwined and often impossible to distinguish because of their common (overlapping) orientations. But method-wise, this exercise may prove fruitful, to bring about many elements or dimensions to how they might highlight a "right to (do) mathematics".

RIGHT TO (DO) MATHEMATICS

Pharmakon

Binary oppositions are not difficult to find when it comes to mathematics. For example, we are familiar with how mathematics is simultaneously accessible and "out of reach". It is something everyone does on a daily basis, but also the craft of some highly trained and exceptional minds. We keep hearing that mathematics are everywhere, but it is also quite the selective discipline (e.g. Jorgensen, Gates, & Roper 2014). Of course, this is "easy to explain": We are talking about different kinds of mathematics, and different institutions in which mathematical activity takes place. We distinguish between "accessible" concepts and others that are more "advanced". More dichotomous concepts come about around this: Simple/complex, easy/difficult, natural/sophisticated, concrete/abstract, practical (applied)/theoretical (pure), and so on. But these dualities are impossible to define in the absolute. A specialist in homotopy might struggle with extremal combinatorics and vice versa, while both would probably be comfortable with functors, which would nevertheless be quite challenging to (most!) primary or secondary school students. At first sight, functors seem more "sophisticated" than fractions, but they can both be seen as general and abstract concepts (being about different kinds of objects). Abstraction, sophistication, usefulness, and complexity are also all subjective and context-dependent terms, and perhaps better designate some of the *directions* in which mathematicians move in their work. Being accessible or "out of reach" are not objective characteristics of mathematical concepts, but relative, situated, contextual features. And so perhaps the "right to mathematics" is not so much about a right to basic or advanced mathematical concepts, but a right to move in such directions.

This is where we touch on the notion of (mathematical) practice(s). Doing mathematics involves navigating between creativity and rigor, intuition and formalism, abstraction and application, certainty and uncertainty, and so on. This comes through various kind of activities, typical of what we *do* when we *do* mathematics: Posing and solving problems, looking for conjectures and proofs, modeling and representing, establishing connections, creating tools and so on. Here, again, these activities can be seen as both available and difficult to get into, especially when we consider them from a first-person perspective. A "problem" is a problem *for someone* and not in itself (e.g. Agre, 1982). Solving 1035÷2 may be quite challenging for a 7-year-old, while proving the Reimann hypothesis might *not* be, as a problem must be something "meaningful" in order to ask it. But it is important to see that mathematics also defines problems, proofs, models or tools in a specific way. Yet, while mathematics might be a "science of pattern", a "study of quantity, structure, space and change", or an "art of logic and reasoning", other fields also do that. Mathematics is what we do when what we do is mathematics (Maheux & Proulx, 2015) and what that precisely is… is difficult (impossible?) to pinpoint.

Then, this is where exclusion is not only a systemic issue, but also clearly takes place in the micropolitics of each and every classroom (e.g. Louie, 2017). Thinking about a right to mathematics in relation to this means embracing the idiosyncratic nature of mathematical activity and its incommensurability from context to context, from person to person. Thus, it may be necessary to reshape institutional structures (including discourses) that hierarchize mathematics and look for ways to organize mathematics education so that all students have the opportunity to engage with mathematics from where they are.

Logocentrism

While we might be tempted to define mathematics as "mathematical knowledge" presented in books, Derrida's notion of *logocentrism* invites us to examine this inclination, and challenge it. Drawing attention to the importance of speech, presence, and written words as forms of meaning further situates the elements discussed above in the actual practice of mathematics. On any given day, tens of millions of people explicitly do mathematics in school or as part of their everyday work (statisticians, engineers, economists, computer scientists, researchers, educators, etc.). Derrida encourages us to acknowledge the various forms of "know-how" inherent to doing mathematics in those diverse contexts, with goals, meanings, and outlooks affecting what is done and how. The act of doing mathematics involves not only the silent manipulation of symbols, but also a dynamic interplay of spoken words, explanations, discussions, and written representations. Whether it be a student solving equations on a chalkboard, an engineer collaborating on a project, or a researcher presenting findings, the mathematical discourse encompasses both spoken and written expressions. And while these different contexts might share some ways of doing mathematics, very often they do not (e.g. FitzSimons 2013).

Derrida's exploration of *logocentrism* challenges us to broaden our understanding of mathematics beyond the confines of written texts, inviting a richer appreciation for the diverse ways in which "mathematical knowledge" is used, conveyed, shared, performed (and constructed) through speech, presence, and the actual actions across various professional and educational settings. It prompts us to recognize that mathematical meaning is not confined to static symbols on a page but instead emerges through the living and dynamic exchange of ideas and expressions. The idea is not to contest the powerful aspect of shared mathematical symbols, notation, and other conventions, but to realize that the "universal" aspect of mathematics suggests a shared, common foundation that enables communication, collaboration, and cooperation among mathematicians, practitioners, educators, researchers, and stakeholders worldwide. This shared background is not a given: We can and must *all* contribute to it, not to build an immutable, certain, absolute, and unanimous basis, but rather to create something radically open, adaptable, and debatable, capable of transforming in response to emerging challenges, opportunities, or innovations within this diverse mathematical community (e.g. Maheux et al. 2019). From there, one can easily imagine how mathematics similarly transforms through its broader interactions with society, culture, economy, environment, technology, ethics, and beyond. A right to mathematics implies the possibility to participate in shaping what counts as mathematics, how, and why. The case of "ethnomathematics" is exemplary here. Conceptualizing mathematics so that it embraces traditional practices (such as weaving) highlights the inherent diversity in mathematical expression asks to *connect with* (and not mearly examine or "include") cultural know-how and ways of doing in Africa or Oceania, but also in Northern Europe (e.g. Tereshkina et al. 2015) for example.

Différance

Derrida's *différance* conceptualizes meaning as disrupting, and challenges essentialist and foundationalist assumptions about stability and certainty. Parallels with the history of mathematics are evident here: We recognize how mathematics transforms over time, not always linearly, but also remains fundamentally "uncertain" (e.g., Klein, 1982). However, this perspective on mathematics and its history is not yet so widespread: Idealism and Platonism may still make their way in people's philosophy of mathematics, highlighting aspects that humanistic views perhaps do not explain as convincingly as one might hope. The history of mathematics is full of examples of "meaning as disrupting", including stories such as those around Zenon's paradoxes or the Pythagorean crisis around the square root of two (echoed by Cantor or Gödel's theorems), and debates around negative or imaginary numbers, for instance. It can also be seen in mathematics' fuzzy relations with other disciplines, as articulated by Châtelet (e.g., 2006) for whom a fundamental part of mathematical activity only takes place when something newly becomes mathematical and unsettls the chain of deductive knowledge (by creating radically new possibilities). A right to mathematics should thus also demand possibilities to create new mathematical ideas, and to work with approximative, provisory, and unsettled notions or assumptions (there have been hundreds, probably even thousands of papers published over the years *assuming* the Riemann Hypothesis).

Considering such ideas in relation with mathematics education is done, for example, by researchers interested in "humanistic mathematics education" (e.g. Brown, 1996). They suggest that doing mathematics involve (and "normalize") confusion, errors, and "good enough" understandings, definitions, solutions, and so on. Différance in signification can also be embraced by dwelling on local idiosyncrasies. For instance, Hewitt and Pimm (2021) recently drew our attention to how fractions have a different meaning in Chinese (language AND mathematics); this illustrates how diversity still exists around very commonly used concepts and despite the ongoing cultural globalization. Less spectacular is the evidence of how, from classroom to classroom, mathematical concepts are defined and understood quite differently. A primary school student generally would not consider multiplication in the same way that a college student learning analysis would. The important point here with the notion of *différance* reminds us that disparities are valuable, inevitable, and essential. A student's mathematical landscape is continuously challenged, with new ideas and possibilities constantly flowing in. Mathematics, from that angle, keeps offering new ways to make sense of the world. Mathematics is an endlessly renewed source of differences that make a difference, as Bateson (1972) put it, in what we see, do, understand, and so on. A right to mathematics is a right to all different kinds of mathematics, and a right to differ from dominant understandings or conceptualizations.

Metaphysics of Presence

To develop Derrida's notion of *metaphysics of presence* in relation to mathematics and mathematics education, I suggest considering how ambiguity is essential to mathematics. While Duval (1999) rightfully draws our attention to how we have "no direct access to mathematical objects but only to their representations" (p. 24), his view on language mostly rests on an idealist epistemology (even referring to Plato) where semiotic marks have a clear referent that "exist" somewhere. The challenge there is to coherently coordinate representations and develop new ones to bring about mathematical ideas. In this view, ambiguity is natural but problematic: We work against it.

Derrida's view takes ambiguity as a condition for language, and challenges the idea that traces (spoken words, symbols, diagrams, etc.) represent some well-defined entities. This includes how traces themselves are never simply there, present, but depend on interpretations shaped by complex interplays between absence, difference, and so on. Ambiguity sparks and feeds the need to engage and try to make present what is absent, to say, to see, to *do* something else than what was said, seen or done. We can easily connect this with Grosholz's (2007) argument about the productive power of ambiguity in mathematics, allowing various theories to interact. Chatelet's (2000) analysis on how new mathematical ideas emerge goes in the same direction; studies have shown how ambiguity also plays an essential role in mundane mathematical thinking (e.g. Gray & Tall, 1991, Beyers, 2007).

When we turn to the mathematics classroom, Barwell (2005) showed how ambiguity is an important feature of "classroom discourse", while Mellone and Tortora (2016) propose to even see it as a cognitive and didactic resource. At the heart of these studies are the observations that ambiguity allows students (and teachers) to discuss and explore, to do mathematics even if they don't have a clear, specific (and shared) understanding or definition, and so on. Powerful mathematical activity takes place precisely when students try to bring about something that is not fully there, when they try to show, to explain, to define, to clarify, to refine, to elaborate, etc. Departing from idealist epistemology also implies rethinking "correctness" in regard to students' mathematical productions. Mathematics education is no longer about "knowledge transposition" to the educational realm and its proper "transmission" to students (as it generally is in the French traditions, for instance). If, instead of being "represented", mathematics emerges from the actual activity that makes it present (Maheux & Proulx, 2015), what matters then are those actual acts, the activity through which students (mathematically) navigate (mathematical) ambiguity. A right to mathematics from that angle calls to embrace and empower students' exploration and engagement in mathematical discourses, leveraging ambiguity as a pathway for meaning-making and to foster a culture of inquiry.

Arche-writing

Arche-writing suggests that the foundational *structures* of language and meaning, or mathematical reasoning and expression, are always already present. Not mathematical objects or ideas, but mathematics as *a way* of thinking and doing that keeps being refined, expanded, clarified, applied, etc. In *Origin of Geometry*, Husserl discusses geometry as a dynamic process of explorations and observations rooted in maturing foundational structures and principles. Indeed, Derrida's famous comment articulates that geometry writes itself through active mathematical work and "on the way to its origin". This implies that mathematical activity emerges from "something already there", and virtually accessible to all individuals (a fundamental aspect of human cognition), but as something malleable, fluid, and adaptable. Yet, while writing itself, mathematics also devises rules and frameworks that participate in the mathematics presently done. They form a fabric to work from or against, a landscape in which something can be mathematically significant. It is thought, from a dynamic configuration of solicited ideas, that meaning, mathematics, and what it means to do mathematics can emerge. *Arche-writing* also stresses the importance of *traces* in this process; traces as a way to revive the (mathematical) activity of others. Fictionalism and socio-cultural epistemologies share some of these tenets.

Cast in a reflection on mathematics education and a right to mathematics, *arche-writing* strongly reminds of Papert's perspective on mathematics, teaching, and learning. Papert stresses the importance of children's culture and everyday life, already infused with mathematics, as a basis for their participation in mathematical activity, and argues that:

> Being a mathematician is no more definable as 'knowing' a set of mathematical facts than being a poet is definable as knowing a set of linguistic facts […] being a mathematician, again like being a poet, or a composer or an engineer, means *doing*, rather than knowing or understanding. (Papert, 1972, p.249)

Inviting students to create in a "microworld" infused with mathematical principles and designed to support mathematical thinking is then a way to put children in a position to *do* mathematics while conceptualizing "knowing [mathematics as] more similar to knowing a person than similar to knowing a fact or having a skill" (Papert, 1980, p. 136). A right to mathematics here has to do with giving students the opportunity to draw on who they are and what they *do* when invited to *do* mathematics, and giving teachers and schools means to do that. Approaching mathematics as an art, as an artistic form and practice, is another way to destabilize the distinction between origin and derivation, and draws attention to the conditions for processes of mathematical meaning-making to occur. Lockhart's (2009) famous *Lament* powerfully makes that connection and illustrates how a right to mathematics should also be right to creatively engage in mathematics, and a right to *write* mathematics.

FIVE ISSUES FOR THE RIGHT TO MATHEMATICS

From the previous section, we see how Derrida's work can relate to social and political dimensions of mathematics education in different ways. The redundant nature of the previous analysis demonstrates that Derrida's ideas are creatively circular, their movement unsettling the taken-for-granted assumptions about what is or should be mathematics and mathematics education. They demand that we ask ourselves tough questions and resist the temptation to simply try and make mathematics education *as we know it* an increasingly more welcoming place. Derrida asks us to go as deep as we can to reflect and debate what we do and why. Putting forward a humanistic, situated, and performative epistemology, his perspective is not neutral, and certainly not "above" potential debates. On the contrary, Derrida steps right into the arena and proposes a vision (of philosophy) from which I have highlighted a few potential features for us: A "right to mathematics"…

- is a right to move in the direction of abstraction, sophistication, usefulness, and so on.
- means embracing the idiosyncratic nature of mathematical activity and reshaping institutional structures that hierarchize mathematics.
- implies the possibility to participate in shaping what counts as mathematics, how, and why.
- demands possibilities to create new (and perhaps provisory) mathematical ideas.
- is a right to all different kinds of mathematics, a right to differ from apparently dominant understandings or conceptualizations.
- calls to foster a culture of inquiry and engagement in mathematical discourses, leveraging ambiguity as a pathway for meaning-making.
- gives opportunities to draw on who we are and what we do, when invited to do mathematics.
- is a right to creatively engage in mathematics.

Concretely implementing such principles is where it is the most difficult, and also where philosophy often falls a bit short. How do we give means to teachers or administrators to revive mathematics education in such a way? How to we engage with them on the significance of such a move? This is where the controversial dimension of Derrida's position plays a part: At the heart of the issue is a need for debates. What matters is not to provoke adhesion, but instead conversations, discussions, and dispute; not consensus, but visible dissensus, disagreement, and discord. As good-hearted as they are, the above features of a right to mathematics are rather vague; figuring out what they could actually imply is precisely something we need to do, and the reason why debates are important. But even more profoundly, the importance of debating what a "right to" means, or involves, is rooted in the conviction what such a right cannot simply be "recognize" or "assigned". It cannot solely exist on paper or in politicians' speeches. An actual right to mathematics can only occur when people live it and organize their lives around it. And doing so requires engaging with the question. In a similarly provocative (and redundant, circular) way, I conclude this paper by formulating five "issues" that emerge, for me, from this analysis of Derrida's ideas. Ahead is the task of figuring out what this really means and entails, and debate on their relevance in relation to a right to mathematics. They are:

- Offering various mathematical experiences and opportunities to teach and learn.
- Conceptualizing different forms of mathematical activities.
- Emphasizing mathematical activity over knowledge or understanding.
- Aiming at developing mathematical familiarity instead of focusing on "expertise".
- Describing mathematics as a living tissue.

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